Effect of Tunnel Pit Radius Variation on the Electric Characteristics of Aluminum Electrolytic Capacitor

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Key factors to the Impedance of Al. E. Cap.

1. The dielectric properties of the anode foil.
2. The pit geometry effect.
3. The Spacer paper and electrolyte system.

Points that had been considered in the presented calculation model

1. The dielectric dispersion based on the linear response function.
2. The pit radius distribution (Statistical Information).
3. The impedance enlargement due to current concentration underneath of the pit opening.

* We neglect the interface capacitance between foil and impregnated paper due to the colloidal dispersion effect in the present¹⁰.

The impedance measurement.

< Equipment >

LCR meter : E4980A (Agilent)

Connection Method : Four – terminal pair method (1m / 50 Ω cable)

Temperature chamber : MC-811T (TABAI)

< Test Specimen of Capacitors >

Rated : 400V, 280 μ F, D25mm x L45mm.

Anode Foil : 120HB15S-580Vf (HEC).

Cathode Foil : 50CK (JCC).

Spacer Paper : PE4-30 (NKK).

Electrolyte : An organic acid ammonium salt, EG solvent, 600 Ω cm at 30 °C.

< Test Condition >

Applied voltage : 0.5Vrms.

Frequency : 20Hz, 60Hz, 100Hz, 120Hz, 200Hz, 300Hz, 360Hz,
500Hz, 1kHz, 2kHz, 5kHz, 10kHz, 20kHz, 50kHz, 100kHz, 200kHz,
500kHz, 1MHz.

Temperature : -40°C, -25 °C, -10 °C, 0 °C, 20°C, 25°C, 40 °C, 60 °C,
85 °C, 105 °C.

Theory and Hypothesis

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1. The dielectric properties expression for the measured Al. E. Cap.

From our another research for the same test capacitors ¹,

$$C(\omega) = \varepsilon_0 \varepsilon_{r0} \frac{S}{d} (\tau_\infty \omega)^{-(1-n)} \Gamma(2-n) \cos \frac{1-n}{2} \pi \quad (1) \quad 1-n(T) = 2.169 \times 10^{-5} T \quad (3)$$

$$\tan \delta = \frac{(\varepsilon_{r0}/\varepsilon_{r\infty})(\tau_\infty \omega)^{-(1-n)} \Gamma(2-n) \sin \frac{1-n}{2} \pi}{1 + (\varepsilon_{r0}/\varepsilon_{r\infty})(\tau_\infty \omega)^{-(1-n)} \Gamma(2-n) \cos \frac{1-n}{2} \pi} \quad (2) \quad \tau_\infty(T) = \tau_\infty(0) \exp[U/T] \quad (4)$$

The parameters:

$$\varepsilon_{r\infty} = 1.006 \quad \varepsilon_{r0} = 9.852$$

$$\varepsilon_0 \frac{S}{d} = 21.72$$

$$U = 2886 \text{ (}^\circ\text{K)}$$

$$\tau_\infty(0) = 2.837 \times 10^{-16} \text{ (sec).}$$

(5)

1. D. Mukaiyama, "Dielectric Characteristics Analysis of Aluminum Electrolyte Capacitors Based on Linear Response Function", Abstract on 235th ECS meeting.

Theory and Hypothesis

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2. The pit radius Distribution and the pit impedance

The impedance of the pit in Fig. 3 is derived as below ².

$$Z_{pit} = e^{-j\frac{\pi}{4}} \sqrt{\frac{\rho_p \ln(1 + d / r_{pit})}{2\pi^2 \omega \epsilon_0 \epsilon_r r_{pit}^2}} \coth \left[e^{j\frac{\pi}{4}} \left(\frac{l_{pit}}{r_{pit}} \right) \sqrt{\frac{2\omega \epsilon_0 \epsilon_r \rho_{el}}{\ln(1 + d / r_{pit})}} \right] \quad (6)$$

Fig. 1. The pit diameter distribution before anodizing (JCC date)

Fig. 2. The anode foil replica cross section.

Fig. 3. The pit model

- The pit radius distribution will be consistent with the log-normal distribution.
- The pit radius will be more significant for the impedance than pit length
- By using Monte-Carlo simulation for pit radius with the parameters : $(r_{pit_mw} \sigma_{pit})$ of the log-normal distribution.
- l_{pit} is fixed to the constant value.

Theory and Hypothesis

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2.1 Determination of the pit density in number : N_{pit} .

Given parameters : l_{pit} , r_{pit_mu} , σ_{pit} , c_a , ρ_{meas} .

c_a : The anode foil capacitance per area (Specified value or measured value).

ρ_{meas} : The electrolyte resistivity of the foil capacitance measurement.

$$Z_{pit_av} = \langle Z_{pit} \rangle \quad (7)$$

$\langle \rangle$ means “Averaged by Monte-Carlo calculation”.

* The r_{pit} series are given as be in accordance with the log-normal distribution (r_{pit_mu} , σ_{pit}) by Monte-Carlo simulation.

$$Z_{bulk} = -j\omega \varepsilon / d \left[1 / N_{pit} - \pi \left\{ (r_{pit_mu} + d)^2 + \sigma_{pit}^2 \right\} \right] \quad (8)$$

$$Z_{anode} = N_{pit} (Z_{pit_av} + Z_{bulk}) \quad (9)$$

The anode foil impedance per area : Z_{anode}

→ N_{pit} can be determined as c_a is consistent with the capacitance derived by eq. (9).

For the derivation of eq. (8), the below property of a log-normal distribution is used.

$$\langle (r_{pit} + d)^2 \rangle = (r_{pit_mu} + d)^2 + \sigma_{pit}^2 \quad (10)$$

➤ N_{pit} is determined based on l_{pit} , r_{pit_mu} , σ_{pit} , c_a , ρ_{meas} .

Fig. 4. The definition of the impedance of the dielerctric of anode foil.

Theory and Hypothesis

2.1 N_{pit} Calculation Flow

Input

* Other input parameters are not showed.

Operation

Judge

Output

2.2 The Basic Structure Concept

To estimate the enlargement effect to the resistance of the electrolyte and spacer paper part by the current concentration underneath of the pit opening, we introduce the basic structure based on the below concept.

- A) The flat part of the dielectric in the basic structure has the same area regardless of the pit radius.
- B) The impedance per area between the anode foil metal layer and the cathode metal layer are represented by the parallel connection of the basic structures per unit area.
- C) The electric field on the cylindric surface of the paper part in the basic structure has only the vertical component (Magnetic Wall Boundary).
- D) The electric field on the top and bottom of the paper part has only the vertical component (Electric Wall Boundary).

Fig. 5. The basic structure

2.2 The Basic Structure Concept

Fig. 5. The basic structure

The surface area of the anode dielectric flat part is given equally the below value to each basic structure due to the hypothesis (A) .

$$1 / N_{pit} - \pi \{ (r_{pit_mu} + d)^2 + \sigma_{pit}^2 \}$$

The radius of the basic structure with the pit radius r_{pit} is obtained by the below equation.

$$\pi r_b^2 = 1 / N_{pit} - \pi \{ (r_{pit_mu} + d)^2 + \sigma_{pit}^2 \} + \pi (r_{pit} + d)^2 \quad (10)$$

The random series of r_{pit} is given by Monte-Carlo based on the log-normal distribution $(r_{pit_mu}, \sigma_{pit})$. Taking the average of the basic structure impedance, we can obtain the averaged impedance of the basic structure following the pit radius distribution.

3. The impedance enlargement factor due to current concentrations underneath of pits.

The solution of Poisson eq. of the electric potential can be solved with taking account of the boundary condition of Fig. 6.

$$\nabla^2 \phi(r, z) = 0 \quad (11)$$

$$\phi(r, z) = \left[1 + 2 \left(\frac{b}{a} \right)^2 \sum_{n=1}^{\infty} \frac{1}{(J_2(x_{1n}))^2} \frac{J_1(x_{1n}b/a)}{x_{1n}b/a} \left\{ \frac{\sinh(x_{1n}z/a)}{(x_{1n}z/a) \cosh(x_{1n}h/a)} J_0(x_{1n}r/a) - 1 \right\} \right] z E_0 \quad (12)$$

The input current is obtained easily.

$$I = (\pi b^2 / \rho_{es}) E_0 \quad (13)$$

The potential of point P is considered as the representative potential, and the impedance is given by

$$Z_{ele1} = \frac{\rho h}{\pi b^2} \left[1 + 2 \left(\frac{b}{a} \right)^2 \sum_{n=1}^{\infty} \frac{1}{(J_2(x_{1n}))^2} \frac{J_1(x_{1n}b/a)}{x_{1n}b/a} \left(\frac{\tanh(x_{1n}h/a)}{x_{1n}h/a} - 1 \right) \right] \quad (14)$$

The impedance between the flat part and the bottom is derived in the same way.

Fig. 6. The paper part of the basic structure, and the boundary condition

3. The impedance enlargement factor due to current concentration underneath of pits.

In the capacitor, these resistances are mixed up. Therefore, we introduce the equivalent circuit as enclosed in red broken line.

$$R_{open} = a_{open} R_{ep} \quad (15) \quad R_{ep} = \frac{\rho_{es} d_p}{\pi r_b^2} \quad (18)$$

$$R_{bulk} = a_{bulk} R_{ep} \quad (16)$$

$$R_c = a_c R_{ep} \quad (17)$$

$$a_{open} = (f_{open} - 1) + \sqrt{(f_{open} - 1)(f_{bulk} - 1)} \quad (19)$$

$$a_{bulk} = (f_{bulk} - 1) + \sqrt{(f_{open} - 1)(f_{bulk} - 1)} \quad (20)$$

$$a_c = 1 - \sqrt{(f_{open} - 1)(f_{bulk} - 1)} \quad (21)$$

$$f_{open} = \left(\frac{r_{pit}}{r_b} \right)^2 + 2 \sum_{n=1}^{\infty} \frac{1}{(J_2(x_{1n}))^2} \frac{J_1(x_{1n} r_{pit} / r_b)}{x_{1n} r_{pit} / r_b} \left(\frac{\tanh(x_{1n} h / r_b)}{x_{1n} h / r_b} - 1 \right) \quad (22)$$

$$f_{bulk} = \frac{2}{1 - (r_{pit} / r_b)^2} \sum_{n=1}^{\infty} \left[\frac{1}{(J_2(x_{1n}))^2} \frac{J_1(x_{1n} r_{pit} / r_b)}{x_{1n} r_{pit} / r_b} \left(\frac{\tanh(x_{1n} h / r_b)}{x_{1n} h / r_b} J_0(x_{1n}) - 1 \right) \right] \quad (23)$$

4. The shunt impedance : Z_{shunt}

The averaged by the impedance of the basic structure is given by Monte-Carlo simulation. And we can obtain the shunt impedance between the anode foil and the cathode foil.

$$z_p = \langle Z_{basic} \rangle / N_{pit} \quad (24)$$

$$Z_{shunt_1} = z_p / W l_1 \quad (25)$$

W : foil width, l_1 : The length from lead tab to the foil edge.

5. Series impedance : z_s

The series impedance from the lead tab to the foil edge was estimated with containing the skin-effect and the temperature effect.

$$Z_{series_1} = \frac{l_1}{W} \left(\frac{\rho_{effect}(\theta, \omega, t_a)}{t_a} + \frac{\rho_{effect}(\theta, \omega, t_c)}{t_c} \right)$$

$$\rho_{effect}(\theta, \omega, t) = \rho_{Al}(\theta) \frac{x}{2} \frac{1 + e^{-x}}{(1 - e^{-x})^2} \quad (26)$$

$$x_{skin} = t / \delta_{skin} \quad (27)$$

$$\rho_{Al}(\theta) = \rho_{Al}(20^\circ C) \{1 + \alpha_{Al}(\theta - 20)\} \quad (28)$$

Fig. 6. The circuit model of the basic structure

6. The impedance of Capacitor

The capacitor part impedance is considered as the transmission line³. And Supposed the lead tab location for the length direction as $l_1 : l_2$, the resistance of lead tab is R_{tabs} , and the parasitic inductance L_{para} . The impedance of capacitor are given by below equations.

$$Z_1 = \sqrt{Z_{series_1} Z_{shunt_1}} \coth \sqrt{\frac{Z_{series_1}}{Z_{shunt_1}}} \quad (29)$$

$$Z_2 = \sqrt{Z_{series_2} Z_{shunt_2}} \coth \sqrt{\frac{Z_{series_2}}{Z_{shunt_2}}} \quad (30)$$

$$Z_{cap} = Z_1 // Z_2 + jL_{para}\omega + R_{tabs} \quad (31)$$

3. R. M. Peekema, J. P. Beesley, Electrochemical Technology, 6, 166-172, 1968.

1. Optimization results of the pit radius distribution parameters r_{pit_mu} , σ_{pit} .

A) The optimized point : $r_{pit_mu} = 0.455\mu\text{m}$ and $\sigma_{pit} = 0.405\mu\text{m}$.

B) Without neglecting the variation : $r_{pit_mu} = 0.37\mu\text{m}$

Fig. 6. The error function of the calculation to the measurement capacitance data at -40°C .

Fig. 7. The comparison of the capacitance. Marks are the measurement data. Solid line is A). Broken line is B)

By taking into account of the variation, the capacitance decreasing by pit geometry are reproduced very well.

2. The effect to the paper part resistance by current concentration underneath of the pit opening

A) $r_{pit_mu} = 0.455\mu\text{m}$ and $\sigma_{pit} = 0.405\mu\text{m}$ (containing this effect)

C) The optimize point : $r_{pit_mu} = 0.35\mu\text{m}$ and $\sigma_{pit} = 0.10\mu\text{m}$ (without this effect)

Fig. 8. The comparison of the capacitance. Marks are the measurement data. Solid line is A). Broken line is C)

Without considering the current concentration, the capacitance decreasing curves tend to have a longer tail.

3. The impedance characteristics comparison with the measurement

A) $r_{pit_mu} = 0.455\mu\text{m}$ and $\sigma_{pit} = 0.405\mu\text{m}$ (containing this effect)

Fig. 9. The comparison of the calculation result with the measurement.

In the range of low temperature and high frequency, the impedance departure from measurement becomes remarkable.

4. The pit radius distribution

Fig. 10. The histogram of the pit diameter. The left-hand side graph corresponds to $r_{pit_mu} = 0.455\mu\text{m}$, $\sigma_{pit} = 0.405\mu\text{m}$, the maximum frequency value is $0.190\mu\text{m}$. The right-hand side graph corresponds to $r_{pit_mu} = 0.35\mu\text{m}$, $\sigma_{pit} = 0.1\mu\text{m}$, the maximum frequency value is $0.31\mu\text{m}$.

Fig. 10. Not considered the effect of eqs. (15) ~ (17).

In the range of low temperature and high frequency, the impedance departure from measurement becomes remarkable.

4. The pit radius distribution

Example of the SEM images of the pit cross section.

Fig. 11. The pit SEM images of the pit cross section of the tunnel pit. The bars length indicates $0.6\mu\text{m}$. The specimens are sampled from the same anode foil ,120HD10-665Vf (JCC).

The pit radius distribution by calculation seems to be in accordance with the real anode foil distribution.

- ◆ Without the pit radius variation, the impedance at low temperature can not be reproduced.
- ◆ The current concentration underneath of the pit opening effects to the tail of the capacitance curve.
- ◆ The impedance departure of the calculation from the measurement is remarkable in the range of low temperature and high frequency. The route of course will be neglecting the colloidal dispersion of the electrolyte and spacer paper system ⁴.
- ◆ The estimated distribution of the pit radius seems to be reasonable.
- ◆ The capacitance change and the dissipation are expressed uniformly on the basis of the response function : $\varphi_0(\tau) = [(1-n)/\tau_\infty](\tau/\tau_\infty)^{-n}$.
- ◆ The dielectric constant compensation between foil measurement and the actual capacitor had to be introduced. Such a factor are generally used in the design process of the Aluminum Electrolytic Capacitors. To complete the simulation of the Aluminum Electrolytic Capacitors, the research of the electric double layer on the interface between the anode foil and the electrolyte will be desired.

4. R. S. Alwitt, Journal of the Electrochemical Society, 116, 1024, 1969.

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Thank You for Your Kind Attention !!